# Relativistic Version of Landau Theory of Fermi Liquid in Presence of Strong Quantizing magnetic Field- An Exact Formalism

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An exact formalism for the relativistic version of Landau theory of Fermi liquid in presence of strong quantizing magnetic field is developed. Both direct and exchange type interactions with scalar and vector coupling cases are considered.

## 1. INTRODUCTION

The study of dense stellar matter under ultra-strong magnetic field has gotten a new life after the discovery of a few magnetars [1–4]. These stellar objects are believed to be strongly magnetized young neutron stars. The surface magnetic fields are observed to be $\geq 10^{15}$G. Then it is quite possible that the field at the core region may go up to $10^{18}$G. The exact source of this strong magnetic field is not known. These objects are also supposed to be the possible sources of anomalous X-ray and soft gamma emissions (AXP and SGR). If the magnetic field is really so strong, in particular at the core region, they must affect most of the important physical properties of such stellar objects. The elementary processes, e.g., weak and electromagnetic processes taking place at the core region should also change significantly.

The strong magnetic field affects the equation of state of dense neutron star matter and as a consequence the gross-properties of neutron stars [5–8], e.g., mass-radius relation, moment of inertia, rotational frequency etc. should change significantly. In the case of compact neutron stars, the phase transition from neutron matter to quark matter at the core region is also affected by strong quantizing magnetic field. It has been shown that a first order phase transition initiated by the nucleation of quark matter droplets is absolutely forbidden if the magnetic field $\sim 10^{15}$G at the core region [9,10]. However a second order phase transition is allowed, provided the magnetic field strength $< 10^{20}$G. This is of course too high to achieve at the core region.

The elementary processes, in particular, the weak and the electromagnetic processes taking place at the core region of a neutron star are strongly affected by such ultra-strong magnetic field [11,12]. Since the cooling of neutron stars are mainly controlled by neutrino/anti-neutrino emission, the presence of strong quantizing magnetic field should affect the thermal history of strongly magnetized neutron stars. Further, the electrical conductivity of neutron star matter which directly controls the evolution of neutron star magnetic field will also change significantly.

In another kind of work, the stability of such strongly magnetized rotating objects are studied. It has been observed from the detailed general relativistic calculation that there are possibility of some form of geometrical deformation of these objects from their usual spherical shapes [13–15]. In the extreme case such objects may either become black strings or black disks. In the non-extreme case, however, it is also possible to detect gravity waves from these rotating objects.

In a recent study of microscopic model of dense neutron star matter, we have observed that if most of the electrons occupy the zeroth Landau level, with spin anti-parallel to the direction of magnetic field and very few of them are with spin along the direction of magnetic field and Landau quantum number $> 0$, then either such strongly magnetized system can not exist or such a strong magnetic field can not be realized at the core region of a neutron star [16]. We have shown in that work that since the electrical conductivity of the medium becomes extremely low in presence of ultra-strong magnetic field, the magnetic field at the core region must therefore decay very rapidly. Hence we have argued that the second possibility is more feasible, i.e., strong magnetic field can not exist at the core of a magnetar.

So far most of the calculations on the equation of states of dense stellar matter in presence of strong magnetic field are either based on some kind of mean field approximation or non-relativistic potential model [6,17]. In this paper we shall derive an exact formalism of relativistic version of Landau theory of Fermi liquid in presence of strong quantizing magnetic field and obtain the quasi-particle energy for both scalar and vector coupling cases. We shall consider a typical many body fermionic system in presence of strong magnetic field interacting via some kind of scalar

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and vector bosons exchange. Therefore from this investigation it is very easy to obtain equation of state of a real fermion system in presence of strong magnetic field, e.g., high density electron gas (with photon exchange), dense neutron star matter (with $\sigma-\omega-\rho$ meson exchange) or even dense quark matter produced at the core region as a result of quark-hadron phase transition (exchange of color gluons). Not only that, one can study the magnetic as well as the non-equilibrium properties of such dense fermionic system as mentioned above. The relativistic version of this model without magnetic field was developed long ago by Baym and Chin [18]. So far most of the scattering or reaction or decay rates/cross-sections are evaluated either with very low magnetic field approximation or in presence of ultra-strong magnetic field. To the best of our knowledge the present article is the first attempt to develop an exact theoretical formalism to obtain the two-body scattering matrix element (both direct and exchange) for all possible strengths of external magnetic field. In a completely different type of work we have used this formalism to obtain the rates of weak and electro-magnetic processes. We have investigated the effect of strong magnetic field on the emissivities and mean free paths of neutrinos / anti-neutrinos and electrical conductivity of dense electron gas in neutron star matter. These quantities play significant role in thermal evolution and the evolution of magnetic fields of neutron stars respectively.

Now similar to the conventional non-relativistic case, the relativistic version also deals with normal Fermi liquid and applicable only for the low-lying excited states of the system, which are made of superpositions of quasi-particle excitations close to the Fermi surface. The relativistic version is therefore applicable to high density electron gas, dense neutron star matter and also to dense quark matter which may present at the core region of compact neutron stars. For the applicability of this model the temperature of the systems are therefore assumed to be low enough compared to the chemical potential of the constituents.

The paper is organized in the following manner. In the next section we shall develop an exact formalism for the relativistic version of Landau theory of Fermi liquid considering a typical many body fermionic system interacting via some kind of scalar and vector boson exchange field. In the last section we have discussed the results and future prospects of the work.

## 2. FORMALISM

We have considered a dense relativistic fermionic system at zero temperature ($T << \mu_f$, the chemical potential of the fermions) and interacting via scalar and vector boson fields represented by $\phi$ and $V^\mu$ respectively with masses $\zeta$ and $\eta$ respectively. The corresponding coupling constants with the fermionic field are $g_s$ and $g_v$ respectively. Then the basic Lagrangian density is given by

$$\begin{aligned}\mathcal{L} &= \frac{i}{2}\bar{\psi}\gamma^\mu(\vec{D}_\mu - \overleftarrow{D}_\mu)\psi - m\bar{\psi}\psi \\ &- g_v\bar{\psi}\gamma^\mu V_\mu\psi - g_s\bar{\psi}\phi\psi + \mathcal{L}_v + \mathcal{L}_s + \mathcal{L}_{\rm em}\end{aligned} \tag{1}$$

Where $D^\mu = \partial^\mu + iqA^\mu$ with $q$ is the magnitude of electric charge, $m$ is the rest mass of the fermions and $A^\mu \equiv (0,0,xB_m,0)$, the electro magnetic field vector. Here the gauge choice is such that the constant external magnetic field $B_m$ is along Z-axis, $\mathcal{L}_v$, $\mathcal{L}_s$ and $\mathcal{L}_{\rm em}$ are the free Lagrangian densities for the vector, scalar and the electromagnetic fields respectively. Therefore, in this particular case, $\mathcal{L}_{\rm em}$ is the Lagrangian density corresponding to the constant magnetic field $B_m$. We now adiabatically turn on the interaction between the fermions. Therefore, the normal Fermi liquid system will evolve from the non-interacting ground state into the interacting ground state and there will be a one-to-one correspondence between the bare particle states of the original system and the dressed or quasi-particle states of the interacting system.

Next to obtain $f$ the Landau Fermi liquid interaction, which is also called the quasi-particle interaction function, we have considered both direct and exchange type interaction. The exchanged bosons are either scalar or vector type. The Landau Fermi liquid interaction $f$ is related to the two-particle forward scattering amplitude. Therefore, it is essential to compute transition matrix element for two-particle direct as well as exchange interaction forward scattering via the exchanged scalar and vector bosons. We further assume that the constant magnetic field acts as a background field and is present throughout the system. We further assume that the strength of magnetic field is such that in the relativistic case the Landau levels are populated for the charged fermions, i.e., $B_m \geq B_m^{(c)(f)}$, the quantum mechanical limit, given by $qB_m^{(c)(f)} = m^2$.

Then the modified form of spinor solutions of charged fermions in presence of strong quantizing magnetic field is given by

$$\psi = \frac{1}{(L_yL_z)^{1/2}}\exp\{-iE_\nu t + ip_y y + ip_z z\}u^{\uparrow\downarrow} \tag{2}$$

where

$$u^{\uparrow} = \frac{1}{[2E_\nu(E_\nu+1)]^{1/2}} \begin{pmatrix} (E_\nu + m) I_{\nu;p_y}(x) \\ 0 \\ p_z I_{\nu;p_y}(x) \\ -i(2\nu q B_m)^{1/2} I_{\nu-1;p_y}(x) \end{pmatrix} \tag{3}$$

and

$$u^{\downarrow} = \frac{1}{[2E_\nu(E_\nu+1)]^{1/2}} \begin{pmatrix} 0 \\ (E_\nu + m) I_{\nu-1;p_y}(x) \\ i(2\nu q B_m)^{1/2} I_{\nu;p_y}(x) \\ -p_z I_{\nu-1;p_y}(x) \end{pmatrix} \tag{4}$$

where the symbols $\uparrow$ and $\downarrow$ indicates up and down spin states.

$$I_\nu = \left(\frac{qB_m}{\pi}\right)^{1/4} \frac{1}{(\nu!)^{1/2}} 2^{-\nu/2} \exp\left[-\frac{1}{2} q B_m \left(x - \frac{p_y}{qB_m}\right)^2\right] H_\nu \left[(qB_m)^{1/2}\left(x - \frac{p_y}{qB_m}\right)\right] \tag{5}$$

with $H_\nu$ the well known Hermite polynomial of order $\nu$, $E_\nu = (p_z^2 + m^2 + 2\nu q B_m)^{1/2}$, the single particle energy eigen value with $\nu = 0, 1, 2, ...$, the Landau quantum numbers and $L_y$, $L_z$ are length scales along $Y$ and $Z$ directions respectively.

In presence of strong quantizing magnetic field along Z-axis (as specified by the choice of gauge) the momentum in the orthogonal plane gets quantized and is given by $p_\perp = (2\nu q B_m)^{1/2}$, whereas, the component along Z-axis varies continuously from $-\infty$ to $\infty$. Further the phase space volume integral in momentum space in this condition is given by

$$\frac{1}{(2\pi)^3}\int d^3p f(p) = \frac{1}{(2\pi)^3}\int dp_z d^2p_\perp f(p) = \frac{qB_m}{4\pi^2}\sum_\nu (2 - \delta_{\nu 0}) \int_{-p_f}^{+p_f} dp_z f(\nu, p_z) \tag{6}$$

here we have assumed $c = \hbar = 0$ and $p_f$ is the Fermi momentum.

To compute the Landau Fermi liquid interaction function $f$ from two particle forward scattering matrix, we first derive from the first principle the general form of two fermion transition matrix element. We calculate for both scalar and vector coupling with direct and exchange type interactions. Therefore, we have the two particle transition matrix corresponding to scalar and vector boson exchange interactions

$$T_{fi} = i\int j^0_{fi}(x)\phi(x) d^4x \tag{7}$$

$$T_{fi} = i\int j^\mu_{fi}(x) V^\mu(x) d^4x \tag{8}$$

where the fermion four-current is given by $j^\mu_{fi}(x) = g_k \bar{\psi}_f(x)\gamma^\mu \psi_i(x) \equiv (\rho(x), \vec{j}(x))$, with the spinors given by eqns.(3) and (4) and $k = s$ and $k = v$ corresponding to scalar and vector coupling cases respectively. The scalar and vector fields can very easily be obtained from the well known Greens function solutions of Klein-Gordon equations

$$(\partial^2 + m^2)\phi, V^\mu = \rho, j^\mu$$

and are given by

$$\phi(x) = -\int \frac{d^4q}{(2\pi)^4} \frac{\exp[-iQ.(x-x')]}{Q^2 - \zeta^2} \rho(x') d^4x' \tag{9}$$

$$V^\mu(x) = -\int \frac{d^4q}{(2\pi)^4} \frac{\exp[-iQ.(x-x')]}{Q^2 - \eta^2} j^\mu(x') d^4x' \tag{10}$$

where $Q$ is the transferred four momentum ($Q^\mu \equiv (q_0, q_x, q_y, q_z)$). Let us first compute the transition matrix element for scalar interaction case. Substituting for the Fermi spinors (eqns.(3) and (4)), we have

$$\rho(x') = \frac{g_s}{L_y L_z} \exp[-i\{(E_{\nu_1} - E_{\nu_2}) - (p_{1y} - k_{1y}) - (p_{1z} - k_{1z})\}][\rho] \tag{11}$$

where

$$[\rho] = \frac{1}{2}\sum_{\text{spin}} \bar{u}(x', \nu_2, k_1)\gamma^0 u(x', \nu_1, p_1) = \frac{1}{2}\text{Tr}(\Lambda(x', \nu_1, \nu_2, p_1, k_1)\gamma^0) \tag{12}$$

Then we have after putting for $\rho(x')$ and integrating over $t'$, $y'$ and $z'$, the direct part of two-particle transition matrix element with scalar coupling interaction

$$\begin{aligned} T_{fi}^{(d,s)} = -i\int \frac{d^4q}{(2\pi)}\delta(q^0 - E_{\nu_1} + E_{\nu_2})\delta(q_y - p_{1y} + k_{1y})\delta(q_z - p_{1z} + k_{1z}) \\ \frac{\exp(-iQ.x)}{Q^2 - \zeta^2}[\rho(x')]\frac{g_s}{L_y L_z}\exp(iq_x x')dx'\frac{g_s}{L_y L_z}[\rho(x)]d^4x \\ \exp[-i\{(E_{\nu_1'} - E_{\nu_2'}) - (p_{2y} - k_{2y}) - (p_{2z} - k_{2z})\}] \end{aligned} \tag{13}$$

Integrating over $t$, $y$, $z$ and $q^0$, $q_y$, $q_z$, we get

$$T_{fi}^{(d,s)} = -i(2\pi)^3\delta(E)\delta(p_y)\delta(p_z)\frac{g_s^2}{L_y^2 L_z^2}\int \frac{dq_x}{2\pi}[\rho(x)][\rho(x')]\exp[-iq_x(x - x')]\frac{1}{Q^2 - \zeta^2}dxdx' \tag{14}$$

where the $\delta$-functions indicate the abbreviated form of energy, $y$ and $z$ components of momentum conservation. In the derivation of eqn.(14) and the computation of subsequent matrix elements for direct case with vector boson exchange, we have assumed the two body process in the form

$$1(p_1) + 2(p_2) \rightarrow 1'(k_1) + 2'(k_2) \tag{15}$$

In this case the forward scattering amplitude can be obtained if we put $E_{\nu_1} = E_{\nu_2} = E_\nu$ (say), $E_{\nu_1'} = E_{\nu_2'} = E_{\nu'}$ (say), $p_{1y} = k_{1y} = p_y$ (say), $p_{2y} = k_{2y} = p_y'$ (say), $p_{1z} = k_{1z} = p_z$ (say) and $p_{2z} = k_{2z} = p_z'$ (say). Then after evaluating the contour integral over $q_x$, given by

$$I_{q_x} = \int_{-\infty}^{+\infty}\frac{dq_x}{2\pi}\frac{1}{Q^2 - \zeta^2} = -\frac{\exp[-\zeta \mid x - x' \mid]}{2\zeta} \tag{16}$$

and substituting in two-body forward scattering matrix element given by eqn.(14), we have the Landau Fermi liquid interaction function corresponding to direct term for scalar coupling case

$$f_{\text{Dir.}}^{(s)} = \frac{g_s^2}{8\zeta}\int \exp[-\zeta \mid x - x' \mid][\rho(x)][\rho(x')]dxdx' \tag{17}$$

After evaluating the quantities within [ ] (see Appendix A) we finally get

$$\begin{aligned} f_{\text{Dir.}}^{(s)} = \frac{g_s^2}{32\zeta E_\nu E_{\nu'}(E_\nu + m)(E_{\nu'} + m)}\int \exp[-\zeta \mid x - x' \mid]dxdx' \\ \{2E_\nu(E_\nu + m)(I_{\nu;p_y}^2(x) + I_{\nu-1;p_y}^2(x)) + 2\nu qB_m(I_{\nu;p_y}^2(x) - I_{\nu-1;p_y}^2(x))\} \\ \{2E_{\nu'}(E_{\nu'} + m)(I_{\nu';p_y'}^2(x') + I_{\nu'-1;p_y'}^2(x')) + 2\nu' qB_m(I_{\nu';p_y'}^2(x') - I_{\nu'-1;p_y'}^2(x'))\} \end{aligned} \tag{18}$$

Then the quasi-particle energy is given by

$$\begin{aligned} \varepsilon_\nu(p_z) &= E_\nu(p_z) + \frac{1}{(2\pi)^2}\sum_{\nu'=0}^{[\nu_{\max}]}(2 - \delta_{\nu'0})\int_{x,x'=-\infty}^{\infty}\int_{p_{y'}=-\infty}^{\infty}\int_{p_{z'}=-p_f}^{p_f} dxdx'dp_{y'}dp_{z'}f_{\text{Dir.}}^{(s)}(p, p'; \nu, \nu') \\ &= E_\nu(p_z) + \Delta E_{\nu;\text{Dir.}}^{(s)}(p_z) \end{aligned} \tag{19}$$

where $[\nu_{\max}] = (\mu_f^2 - m^2)/(2qB_m)$, the integer part of right hand side. In the case of ultra-strong magnetic field, $\nu = \nu' = 0$, then

$$\varepsilon_0(p_z) = E_0(p_z) + \frac{g_s^2}{\zeta^2} n_f \tag{20}$$

where

$$n_f = \frac{qB_m}{2\pi^2} p_f \tag{21}$$

the number density of fermions. The form of the result given in eqn.(20) is identical with that of zero field case [18]. The energy density is then given by

$$\epsilon = \frac{qB_m}{4\pi^2} \sum_{\nu=0}^{[\nu_{\max}]} (2-\delta_{\nu 0}) \int_{-p_f}^{p_f} dp_z \varepsilon_\nu(p_z) \tag{22}$$

Let us now compute the exchange part for scalar coupling case. In this case we substitute $E_{\nu_1} = E_{\nu_2'} = E_\nu$, $E_{\nu_1'} = E_{\nu_2} = E_{\nu'}$, $p_{1y} = k_{2y} = p_y$, $p_{1z} = k_{2z} = p_z$, $p_{2y} = k_{1y} = p_y'$, $p_{2z} = k_{1z} = p_z'$, to obtain the two-body forward scattering amplitude. In this particular example, the basic process is

$$1(p_1) + 2(p_2) \rightarrow 1'(k_2) + 2'(k_1) \tag{23}$$

and the $q_x$ contour integral is given by

$$I_{q_x} = -\frac{\exp[-K \mid x - x' \mid]}{2K} \tag{24}$$

where $K \approx (q_y^2 + q_z^2 + \zeta^2)^{1/2}$ and $q_i = (p_i - p_i')$, with $i = y$ and $z$. Then evaluating $[\rho(x)\rho(x')]$ (see Appendix A), we get

$$\begin{aligned} f_{\rm ex}^{(s)} = & \frac{g_s^2 m}{32 E_\nu E_{\nu'}} \int dx dx' \frac{\exp[-K \mid x - x' \mid]}{K} \\ & [(E_\nu E_{\nu'} + p_z p_z' + m^2) \mathrm{Tr}(AA') - \vec{p}_\perp . \vec{p}_\perp' \mathrm{Tr}(BB')] \end{aligned} \tag{25}$$

where

$$\mathrm{Tr}(AA') = 2(I_\nu(x) I_\nu(x') I_{\nu'}(x) I_{\nu'}(x') + I_{\nu-1}(x) I_{\nu-1}(x') I_{\nu'-1}(x) I_{\nu'-1}(x')) \tag{26}$$

and

$$\mathrm{Tr}(BB') = 2(I_{\nu-1}(x) I_\nu(x') I_{\nu'-1}(x) I_{\nu'}(x') + I_\nu(x) I_{\nu-1}(x') I_{\nu'}(x) I_{\nu'-1}(x')) \tag{27}$$

Then as before

$$\Delta E_{\nu;\rm ex}^{(s)} = \frac{1}{(2\pi)^2} \sum_{\nu'=0}^{[\nu_{\max}]} (2-\delta_{\nu' 0}) \int_{x,x'=-\infty}^{\infty} \int_{p_{y'}=-\infty}^{\infty} \int_{p_{z'}=-p_f}^{p_f} dx dx' dp_{y'} dp_{z'} f_{\rm ex}^{(s)}(p, p'; \nu, \nu') \tag{28}$$

For $\nu = \nu' = 0$,

$$\begin{aligned} \Delta E_{\nu;\rm ex}^{(s)} = & \frac{g_s^2}{8\pi^2} \int_{-p_f}^{p_f} dp_z' \exp\left(\frac{v'^2}{qB_m}\right) \frac{(E_0 E_0' + p_z p_z' + m^2)}{E_0 E_0'} \\ & \int_0^{\pi/2} \mathrm{erfc}\left(\frac{v'}{(2qB_m)^{1/2}} \sec\theta\right) \sec\theta d\theta \end{aligned} \tag{29}$$

where $v' \approx \mid p_z - p_z' \mid$. In the zero field case

$$f_{\rm ex}^{(s)} = \frac{g_s^2}{4E_0(p) E_0(p')} \frac{m^2 - p.p'}{(p'-p)^2 + \zeta^2} \tag{30}$$

We next consider the vector coupling case. The current four vector in the direct case is given by

$$j^\mu(x) = g_v \frac{\exp[-i\{(E_{\nu_1} - E_{\nu_2}) - (p_{1y} - k_{1y}) - (p_{1z} - k_{1z})\}]}{L_y L_z}[j^\mu(x, p_1, k_1)] \tag{31}$$

where

$$[j^\mu] = \frac{1}{2}\sum_{\rm spin} \bar{u}\gamma^\mu u = \frac{1}{2}{\rm Tr}(\Lambda(x, \nu_1, \nu_2, p_1, k_1)\gamma^\mu) \tag{32}$$

Then it is very easy to show that the Landau Fermi liquid interaction function is given by

$$f_{\rm Dir}^{(v)} = \frac{g_v^2}{32\eta E_\nu E_{\nu'}} \int dx dx' \exp[-\eta \mid x - x' \mid][(E_\nu E_{\nu'} - p_z p_{z'}){\rm Tr}A\ {\rm Tr}A' + \vec{p}_\perp.\vec{p}'_\perp {\rm Tr}B\ {\rm Tr}B'] \tag{33}$$

where we have used the same kind of substitutions as has been done for the scalar case and further the relation,

$${\rm Tr}(\Lambda(x, \nu, p)\gamma^\mu) = \frac{1}{2E_\nu}({\rm Tr}A + {\rm Tr}B)p^\mu \tag{34}$$

(see Appendix B) is used to obtain this expression.

The exchange term corresponding to vector coupling case is similarly given by (see Appendix B for brief derivation)

$$f_{\rm Ex}^{(v)} = \frac{g_v^2}{32 E_\nu E_{\nu'}} \int dx dx' \frac{\exp[-K \mid x - x' \mid]}{K}[(p_z p_{z'} - E_\nu E_{\nu'} + 3m^2){\rm Tr}(AA') - \vec{p}_\perp.\vec{p}'_\perp {\rm Tr}(BB')] \tag{35}$$

Then energy density of the system is given by (including all the terms)

$$\epsilon = \frac{qB_m}{4\pi^2} \sum_{\nu=0}^{[\nu_{\rm max}]} (2 - \delta_{\nu 0}) \left\{ \int_{-p_f}^{+p_f} dp_z E_\nu(p_z) + \sum_{k,l} \int_{-p_f}^{+p_f} dp_z \Delta E_{\nu;(l)}^{(k)}(p_z) \right\} \tag{36}$$

where $k = s$ or $v$ and $l =$ Dir or ex. The kinetic pressure is then given by

$$P = \mu_f n_f - \epsilon \tag{37}$$

The chemical potential or the Fermi energy is given by

$$\begin{aligned} \mu_f = \varepsilon_\nu(p_z = p_f) &= E_\nu(p_f) + \sum_{k,l} \Delta E_{\nu;(l)}^{(k)}(p_f) \\ &= \mu_0 + \Delta\mu \end{aligned} \tag{38}$$

Therefore knowing the modified form of single particle energy or the quasi-particle energy, it is possible to obtain all the thermodynamic quantities of the system from standard definitions.

To obtain equation of state of dense neutron star matter or dense electron gas, since both direct and exchange terms contribute in the case of $e + e \to e + e$ scattering, it is necessary to screen $e - e$ and $e - p$ interactions in the direct case (in the case of $e - p$ scattering, of course, only the direct term exists). To get the screening mass of electron, we proceed in the following manner. We assume protons as the positively charged background. Then assuming local charge fluctuation we have

$$\begin{aligned} n^+ &= n_0^+ + n_1^+ \\ n_e &= n_0^+ + n_e' \end{aligned} \tag{39}$$

In the case positive lattice background, we have to replace $n_0$ by $Zn_0$, where $Z$ is the atomic number of the lattice ions. Then in the case of proton background, the well known Poisson equation is given by

$$\nabla^2\psi = -4\pi n_1^+ e + 4\pi n_e' e \tag{40}$$

where $\psi$ is the electrostatic potential produced in the system because of local charge fluctuation. Then we have from eqn.(21) after substituting for $p_f$ in terms of chemical potential $\mu_f$ and electrostatic potential $\psi$,

$$n_e = n_0^+ + \frac{eB_m}{2\pi^2}\sum_\nu (2 - \delta_{\nu 0}) \frac{\mu_f e\psi}{(\mu_f^2 - m^2 - 2\nu e B_m)^{1/2}} \tag{41}$$

The last term is the perturbed part ($n_e'$) as mentioned in the second line of eqn.(39). Then we have

$$\nabla^2\psi = -4\pi n_1^+ e + \frac{2e^2 B_m \mu_f}{\pi}\sum_\nu (2-\delta_{\nu 0})\frac{1}{(\mu_f^2 - m^2 - 2\nu e B_m)^{1/2}} \tag{42}$$

which may be re-written in the form

$$\left[\nabla^2 - k_{\rm sc}^2\right]\psi = -4\pi n_1^+ e \tag{43}$$

where $k_{\rm sc}$ is the screening mass and is given by

$$k_{\rm sc} = \left[\frac{2e^2 B_m \mu_f}{\pi}\sum_\nu (2-\delta_{\nu 0})\frac{1}{(\mu_f^2 - m^2 - 2\nu e B_m)^{1/2}}\right]^{1/2} \tag{44}$$

and the screening length $r_D = 1/k_{\rm sc}$

In the case of quark matter, however, only the exchange term contributes, we therefore do not need quark-quark screening. Since colorless gluons do not exist, the direct term has no significance.

## 3. CONCLUSION

In this article we have developed an exact formalism to obtain two-body scattering matrix element for an wide range of magnetic field strength. Although the result has been used to formulate an exact relativistic version of Landau theory of Fermi liquid, is also applicable to study weak and electromagnetic processes in presence of strong magnetic field.

The formalism developed in this paper to compute Landau Fermi liquid interaction $f$ can also be used to study magnetic properties of fermionic system. The calculations (both theoretical and numerical) are of course quite complicated in the relativistic region (we shall report the result in some future publication). The other interesting property- the transport theory of normal Fermi liquid can very easily be studied with the help of present formalism and obtain various kinetic coefficients along with their dependences on the strength of magnetic field.

## APPENDIX A

In the direct case we need

$$\Lambda(\nu, x, x', k_y) = \sum_{\rm spin} u(\nu, x, k)\bar{u}(\nu, x', k) \tag{A1}$$

Substituting for the up and down spin solutions of Dirac equation (eqns.(3) and (4)), we get

$$\Lambda = \frac{1}{2E_\nu}(Ak_\mu\gamma^\mu(\mu = 0 \ \text{ and } \ z) + mA + Bk_\mu\gamma^\mu(\mu = y \ \text{ and } \ p_y = p_\perp)) \tag{A2}$$

The matrices $A$ and $B$ are given by

$$A = \begin{pmatrix} I_\nu I_\nu' & 0 & 0 & 0 \\ 0 & I_{\nu-1}I_{\nu-1}' & 0 & 0 \\ 0 & 0 & I_\nu I_\nu' & 0 \\ 0 & 0 & 0 & I_{\nu-1}I_{\nu-1}' \end{pmatrix} \tag{A3}$$

$$B = \begin{pmatrix} I_{\nu-1}I_\nu' & 0 & 0 & 0 \\ 0 & I_\nu I_{\nu-1}' & 0 & 0 \\ 0 & 0 & I_{\nu-1}I_\nu' & 0 \\ 0 & 0 & 0 & I_\nu I_{\nu-1}' \end{pmatrix} \tag{A4}$$

where the primes indicate the functions of $x'$.

Eqn.(A2) is an entirely new result and to the best of our knowledge it has not been reported earlier. Further, this result plays the key role in all kind of calculations related with electromagnetic and weak interactions in presence of strong quantizing magnetic field.

Since $\gamma$ matrices are traceless and both $A$ and $B$ matrices are diagonal with identical blocks, it is very easy to evaluate the traces of the product of $\gamma$-matrices multiplied with any number of $A$ and/or $B$, from any side with any order e.g.,

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu A_1A_2..B_1B_2..) = Tr(A_1A_2..B_1B_2..)g^{\mu\nu}, \tag{A5}$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma A_1A_2..B_1B_2..) = Tr(A_1A_2..B_1B_2..)(g^{\mu\nu}g^{\sigma\lambda} - g^{\mu\lambda}g^{\nu\sigma} + g^{\mu\lambda}g^{\nu\sigma}), \tag{A6}$$

Tr(product of odd $\gamma$s with $A$ and/or $B$) $= 0$ etc. The other interesting aspects of $A$ and $B$ matrices are:
i) $k_{1\mu}k^{2\mu}\mathrm{Tr}(A_1A_2) = (E_1E_2 - k_{1z}k_{2z})\mathrm{Tr}(A_1A_2)$
ii) $k_{1\mu}k^{2\mu}\mathrm{Tr}(B_1B_2) = \vec{k}_{1\perp}.\vec{k}_{2\perp}\mathrm{Tr}(B_1B_2)$
iii) $k_{1\mu}k^{2\mu}\mathrm{Tr}(A_1B_2) = k_{1\mu}k^{2\mu}\mathrm{Tr}(B_1A_2) = 0$
iv) $p_{1\mu}k^{1\mu}p_{2\nu}k^{2\nu}\mathrm{Tr}(A_1B_2) \neq 0 = (E_{\nu_1}E_{\nu_2'} - p_{1z}k_{1z})\vec{p}_{2\perp}.\vec{k}_{2\perp}\mathrm{Tr}(A_1B_2)$ These set of relations are also totally new results and have not been reported before.

To compute the direct part for scalar coupling case, we need

$$\mathrm{Tr}(\bar{u}(x,\nu,p)u(x,\nu,p)\gamma^0) = \mathrm{Tr}(\Lambda(x,\nu,p)\gamma^0) \tag{A7}$$

Using the expression for $\Lambda$, it very easy to show that the above trace is given by

$$\frac{1}{2E_\nu(E_\nu+m)}[2E_\nu(E_\nu+m)(I^2_{\nu;p_y}(x) + I^2_{\nu-1;p_y}(x)) + 2\nu qB(I^2_{\nu;p_y}(x) - I^2_{\nu-1;p_y}(x))] \tag{A8}$$

To compute the exchange part for this case, we need

$$\mathrm{Tr}[(\bar{u}(x,\nu,p)\gamma^0u(\nu',x,p'))(\bar{u}(x',\nu',p')\gamma^0u(\nu,x',p))] = \mathrm{Tr}[\Lambda(x,x',p,\nu)\gamma^0\Lambda(x,x',p',\nu')\gamma^0] \tag{A9}$$

Then by simple algebraic manipulation it is trivial to show that the above trace is given by

$$\frac{1}{4E_\nu E_{\nu'}}[(E_\nu E_{\nu'} + p_zp_z' + m^2)\mathrm{Tr}(AA') - \vec{p}_\perp.\vec{p}_\perp'\mathrm{Tr}(BB')] \tag{A10}$$

where

$$\mathrm{Tr}(AA') = 2(I_\nu(x)I_\nu(x')I_{\nu'}(x)I_{\nu'}(x') + I_{\nu-1}(x)I_{\nu-1}(x')I_{\nu'-1}(x)I_{\nu'-1}(x')) \tag{A11}$$

$$\mathrm{Tr}(BB') = 2(I_{\nu-1}(x)I_\nu(x')I_{\nu'-1}(x)I_{\nu'}(x') + I_\nu(x)I_{\nu-1}(x')I_{\nu'}(x)I_{\nu'-1}(x')) \tag{A12}$$

## APPENDIX B

Now to compute two-particle forward scattering matrix element for vector boson exchange case we need

$$\mathrm{Tr}(u(x,\nu,p)\bar{u}(x,\nu,p)\gamma^\mu) \tag{B1}$$

for direct case. It is trivial to show that the trace is given by

$$\frac{1}{2E_\nu}\mathrm{Tr}[(Ap_\sigma\gamma^\sigma + mA + Bp_\sigma\gamma^\sigma)\gamma^\mu] = \frac{1}{2E_\nu}[p^\mu(\mathrm{Tr}A + \mathrm{Tr}B)] \tag{B2}$$

Then

$$[j^\mu(x)][j_\mu(x')] = \frac{1}{4E_\mu E_{\mu'}}[(E_\nu E_{\nu'} - p_zp_z')\mathrm{Tr}A\mathrm{Tr}A' + p_\perp p_\perp'\mathrm{Tr}B\mathrm{Tr}B'] \tag{B3}$$

In this case the terms $\mathrm{Tr}A\mathrm{Tr}B'$ and $\mathrm{Tr}A'\mathrm{Tr}B$ do not contribute. In the exchange interaction for vector coupling case, we need

$$\mathrm{Tr}[\Lambda(x,x',p,\nu)\gamma^\mu\Lambda(x,x',p',\nu')\gamma^\mu] \tag{B4}$$

By some trivial algebra, we can very easily show that this expression is given by

$$\frac{1}{4E_\nu E_{\nu'}}\mathrm{Tr}[AA'p_\sigma p'_\lambda(\gamma^\sigma\gamma^\mu\gamma^\lambda\gamma_\mu)+m^2AA'\gamma^\mu\gamma_\mu+p_\sigma p'_\lambda BB'(\gamma^\sigma\gamma^\mu\ \gamma^\lambda\gamma_\mu)] \tag{B5}$$

Then using the well known formula for the product of four $\gamma$-matrices, we have the final form of the above expression

$$\frac{1}{4E_\nu E_{\nu'}}[(p_zp'_z-E_\nu E_{\nu'})\mathrm{Tr}(AA')+3m^2\mathrm{Tr}(AA')-\vec{p}_\perp.\vec{p}'_\perp\mathrm{Tr}(BB')] \tag{B6}$$

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